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BENEFIT OF CONSTANT MOMENTUM PROPULSION FOR LARGE ΔV MISSIONS – APPPLICATION IN LASER PROPULSION*

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ABSTRACT

We show that perfect propulsion requires a constant momentum mission, as a consequence of Newton's second law. Perfect propulsion occurs when the velocity of the propelled mass in the inertial frame of reference matches the velocity of the propellant jet in the rocket frame of reference. We compare constant momentum to constant specific impulse propulsion, which, for a given specification of the mission delta V, has an optimum specific impulse that maximizes the propelled mass per unit jet kinetic energy investment. We also describe findings of more than 50% efficiency for conversion of laser energy into jet kinetic energy by ablation of solids.

INTRODUCTION

Laser propulsion has potential advantages over chemical propulsion because the energy source and propellant are separated from each other.[1,2] In principle, very large amounts of energy can be forced into relatively small amounts of propellant mass. The specific internal energy of the propellant, Q* energy per mass, may be regulated to control specific impulse.[3] Propellant jet exit velocity (v_{jet}, in the rocket frame of reference) may be tailored to match vehicle velocity (v, in inertial frame of reference); throughout the mission $v = v_{jet}$. The maximum possible accelerated mass (m, payload) per unit of jet kinetic energy (E_{jet}) results when $v = v_{jet}$ because the jet kinetic energy transfers completely into the vehicle. Propulsion at constant specific impulse i.e., constant v_{jet}, produces a loss of payload mass because the propellant slips in the inertial frame.

This paper sets forth elementary relationships to quantify the payload advantage of a $v = v_{jet}$ mission over a fixed v_{jet} mission.

In principle, with pulsed laser propulsion, the time for energy addition to propellant may be controlled to be less than the time for hydrodynamic expansion of heated propellant.[4] In the limit where energy addition occurs at constant volume, the propellant is confined by its own inertia during heating, and the initial propellant density is conserved. The extremely high pressure achievable in this limit for condensed phase propellant (density > 1000 kg/m^3, as in laser ablation) enables conversion of a larger fraction, α, of propellant internal energy into jet kinetic energy with relatively smaller expansion ratio.

The thermodynamics of isentropic expansion may be used to establish upper limits to α for blow down expansion of laser heated propellant from a constant volume with known initial Q* and density, as we described previously for laser heated air.[5,6]

When laser energy is the limiting factor, the figure of merit, m/E_L, the accelerated mass (accelerated by Δv) per unit of laser energy invested, assumes a dominant role. Maximization of m/E_L requires a good understanding of energy absorption, its conversion to jet kinetic energy, and coupling of jet kinetic energy into the vehicle. In conventional propulsion, the mass fraction, the accelerated mass per unit of initial mass investment, plays the dominant role.

INSTANTANEOUS PROPULSION EFFICIENCY

One of the earlier mentions of propulsion efficiency appears in the 1949 edition of Sutton's textbook[7], where the instantaneous propulsion efficiency, η_i, which is defined as the fraction of jet kinetic energy that is converted into vehicle

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kinetic energy, is expressed in terms of the vehicle velocity in the inertial frame of reference, v, and the jet velocity in the rocket frame of reference, v_{jet},

$$\eta_i = \frac{2\,(v/v_{jet})}{1+(v/v_{jet})^2}. \qquad (1)$$

Thus, when $v \neq v_{jet}$, $\eta < 1$. When the jet is left behind with a non-zero velocity in the inertial frame, some of its kinetic energy is wasted. Perfect kinetic energy transfer from jet to vehicle occurs only when $v = v_{jet}$, such that the jet is deposited with zero velocity in the inertial frame.

CONSTANT MOMENTUM COMPARED TO CONSTANT SPECIFIC IMPULSE MISSION

The Constant Specific Impulse Mission

The rocket equation is obtained for a constant specific impulse (constant v_{jet}) mission by integration of Newton's second law.[8] From $F = -d(mv)/dt$, one may obtain $mdv = -v_{jet}dm$ by neglecting gravity and drag forces. Thus,

$$\int_{m_o}^{m} \frac{dm}{m} = \frac{-1}{v_{jet}} \int_{v_o}^{v} dv, \qquad (2)$$

which integrates to produce the rocket equation,

$$f = \frac{m}{m_o} = \exp -\left(\frac{v - v_o}{v_{jet}}\right) = \exp \frac{-\Delta v}{v_{jet}}, \quad (3)$$

where the mass fraction, f, the ratio of the propelled mass, m, to the initial mass, m_o, is a function of only the ratio $\Delta v/v_{jet} = x$, so $f = e^{-x}$, with Maclaurin expansion $1/f = 1 + x + x^2/2! + x^3/3! + \dots$.

The Constant Momentum Mission

Application of Newton's second law to the mission where $v_{jet} = v$ yields,

$$\int_{m_o}^{m} \frac{dm}{m} = -\int_{v_o}^{v} \frac{dv}{v}. \qquad (4)$$

Eq. 4 produces another rocket equation for a constant momentum mission, $m_o v_o = mv$, and

$$f' = \frac{m}{m_o} = \frac{v_o}{v} = 1 - \frac{\Delta v}{v} = \left(1 + \frac{\Delta v}{v_o}\right)^{-1}, \qquad (5)$$

where f' denotes the mass fraction of a constant momentum mission.

Figures of Merit for Laser Propulsion: m/E_{jet}

The figures of merit for constant specific impulse and constant momentum missions are denoted B and B' respectively. They represent the amount of mass that is accelerated by a velocity increment (Δv) per unit of jet kinetic energy invested. For the constant specific impulse mission Eq. 3 applies, $f = e^{-x}$, so after using mass m_o-m of propellant,

$$E_{jet} = -\tfrac{1}{2}\int_{m_o}^{m} v_{jet}^2\, dm = \tfrac{1}{2}(m_o - m)\, v_{jet}^2, \quad (6)$$

$$B = \frac{m}{\tfrac{1}{2}(m_o - m)\, v_{jet}^2} = \frac{2x^2}{(e^x - 1)[\Delta v]^2} = \frac{2f(\ln f)^2}{(1-f)[\Delta v]^2} \qquad (7)$$

During the constant momentum mission ($mv = m_o v_o$) the jet exit velocity increases so we must do integration of variable v^2 over the propellant mass to obtain the kinetic energy invested in the jet, viz.

$$E'_{jet} = -\tfrac{1}{2}\int_{m_o}^{m} v^2 dm = -\tfrac{1}{2}(m_o v_o)^2 \int_{m_o}^{m} \frac{dm}{m^2}$$

$$= \tfrac{1}{2}mv^2 - \tfrac{1}{2}m_o v_o^2 = \tfrac{1}{2}mv\Delta v \qquad (8)$$

which confirms perfect kinetic energy transfer from jet to vehicle. Eq. 8 shows that the kinetic energy gain, $\frac{1}{2}mv^2 - \frac{1}{2}m_o v_o^2$, is the final vehicle kinetic energy minus the kinetic energy of the initial mass, which is composed of vehicle and propellant. Thus, for the constant momentum mission,

$$B' = \frac{m}{\tfrac{1}{2}mv\Delta v} = \frac{2(1-f')}{[\Delta v]^2} \qquad (9)$$

Fig. 1 shows plots of the dimensionless quantities, $\frac{1}{2}\Delta v^2 B$ and $\frac{1}{2}\Delta v^2 B'$ as a function of the respective mass fractions, $f = \exp(-\Delta v/v_{jet})$ and f' =

$1 - \Delta v/v = 1/(1+\Delta v/v_o)$. In the limit, as the mass fraction approaches zero, m/E_{jet} approaches $2/[\Delta v]^2$ for the constant momentum case but approaches zero for the constant specific impulse case. For the constant specific impulse case, $\frac{1}{2}\Delta v^2 B$ shows a maximum of 0.647365 at f = 0.203179, or $\Delta v/v_{jet}$ = - lnf= 1.59367. The maximum may be calculated exactly by differentiation of Eq. 7, setting dB/df = 0, and solving the resulting transcendental equation for the maximum of B and f. Two independent discussions[9,10] of this aspect of constant specific impulse propulsion appeared 27 years ago; further elaboration has appeared in more recent publications on laser propulsion.[11,12] Here we have shown that B_{max} occurs at $\Delta v/v_{jet}$ = 1.59367, and

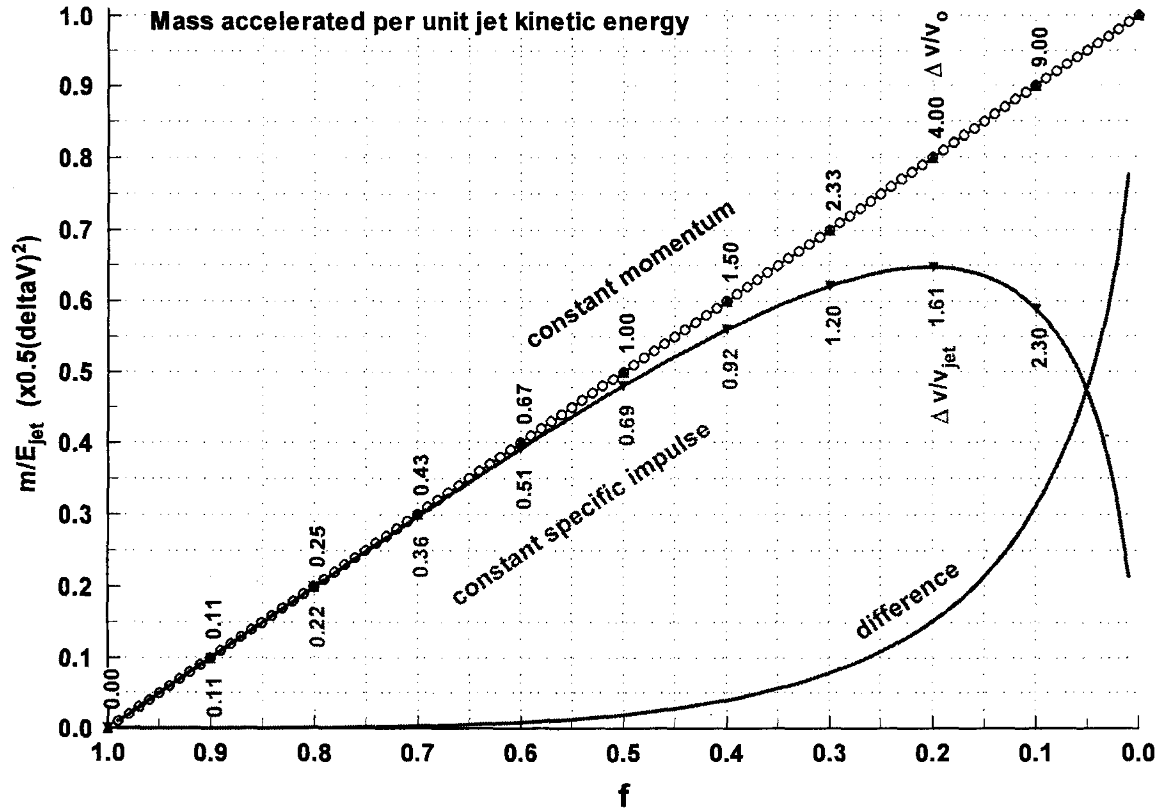

Figure 1. Dimensionless figure of merit for propulsion, m/E_{jet} [$\frac{1}{2}(\Delta v)^2$] for constant momentum and constant specific impulse propulsion. Values of $\Delta v/v_{jet}$ = - ln f for constant specific impulse mission and $\Delta v/v_o$ = (1-f)/f for constant momentum missions are shown. Mission requirements to achieve the maximum $m/E_{jet} = 0.647365/[\frac{1}{2}(\Delta v)^2]$ for the constant specific impulse mission are f = 0.203179 or $\Delta v/v_{jet}$ = 1.59367, and t (mission time) = $0.313736[m_o(\Delta v)^2/2P_{jet}]$.

that the maximum depends only on Δv and is independent of the initial velocity. The maximum exists regardless of the choice of inertial frame. Also, as will be shown below, the time to accomplish an optimized constant specific impulse mission is a certain fraction of the ratio of $m_o(\Delta v)^2$ to the jet power, which is another consequence of Newton's second law.

An effective Δv of about 10,000 m/s is required to launch from Earth into low earth orbit, accounting for about 1000 m/s each of gravity and drag losses. The optimized constant specific impulse mission orbits a maximum of 0.0129 kg of mass per MJ of jet kinetic energy and requires f = 0.203, or v_{jet} = 6275 m/s (Isp = 640 s). At this mass fraction, the constant momentum mission produces $\frac{1}{2}\Delta v^2 B'$ = 0.797 and orbits 0.0159 kg per MJ, about 23% more mass per unit jet energy than the constant specific impulse launch to LEO. Also, the constant momentum mission orbits nearly twice as much mass, f' = 0.35 as compared to f = 0.20 for the same $m/E_{jet} = 0.647[(\Delta v)^2/2]$. At high mass fraction, greater than f ~ 0.6, the two mission types become indistinguishable. In the limit as f approaches unity (or as $\Delta v/v_{jet}$ approaches zero), the mass fraction of the constant

specific impulse mission approaches that of the constant momentum mission, viz., $1/f = e^x = 1 + x + x^2/2! + ...$, or $f = (1 + \Delta v/v_{jet})^{-1}$, which has the same form as Eq. 5.

Mission Time for constant momentum and constant specific impulse missions

The kinetic power in the jet determines the time required for achievement of a specified Δv with a specified initial mass and specific impulse. The jet power is

$$P_{jet} = \tfrac{1}{2} v_{jet}^2 \frac{dm}{dt} = \tfrac{1}{2} F v_{jet}. \qquad (10)$$

Integration of $dm/dt = 2P_{jet}/v_{jet}^2$ produces

$$f = \frac{m}{m_o} = 1 - \frac{2P_{jet}}{m_o v_{jet}^2} t. \qquad (11)$$

The mission time may be written explicitly in terms of mission parameters, Δv, m_o, P_{jet} and f:

$$t = \frac{m_o}{2P_{jet}} (\Delta v)^2 \frac{(1-f)}{(\ln f)^2} \qquad (12)$$

The optimum time for a constant specific impulse mission is thus obtained from f (optimum) = 0.203179 and Eq. 12 as t = 0.313736 $[m_o/2P_{jet}](\Delta v)^2$. Thus for a rocket that has initial mass of 20 kg and jet power of 1 MW, the optimum mission time is 313.736 seconds for Δv of 10,000 m/s launch to LEO.

Substitution of f from Eq. 11 into Eq. 3 produces three equivalent expressions for Δv:

$$\Delta v = -v_{jet} \ln\left(1 - \frac{2P_{jet}}{m_o v_{jet}^2} t\right)$$

$$= \sqrt{\frac{2P_{jet}}{m_o} \frac{(\ln f)^2}{(1-f)} t}$$

$$= \ln\left(\frac{BP_{jet}}{m_o} t\right) \sqrt{\frac{\frac{2P_{jet}}{m_o}}{\left(1 - \frac{BP_{jet}}{m_o} t\right)} t} \qquad (13)$$

where the last equality may be obtained from the definition of $B = m/E_{jet} = fm_o/P_{jet}t$ and substitution of $f = BP_{jet}t/m_o$ into the preceeding equality.

For the constant momentum mission we may use Newton's law to get the vehicle velocity as a function of time, viz., $F = mdv/dt = 2\ P_{jet}/v_{jet}$, $v = v_{jet}$, $m_o v_o = mv$, and $f' = B' P_{jet} t/m_o$ to obtain three equivalent expressions for the velocity increment ($\Delta' v$) of a constant momentum mission:

$$\Delta' v = \frac{2P_{jet}}{m_o v_o} t'$$

$$= \sqrt{\frac{2P_{jet}}{m_o} \frac{(1-f')}{f'} t'}$$

$$= \sqrt{\frac{2}{B'} - \frac{2P_{jet}}{m_o} t'} \qquad (14)$$

Then, Eq. 5 may be used with $\Delta' v/v_o = 2P_{jet}t/m_o v_o^2$, which comes from the first equality of Eq. 14, to obtain the mass fraction for the constant momentum mission as a function of time:

$$f' = \frac{m}{m_o} = \left[1 + \frac{2P_{jet}}{m_o v_o^2} t'\right]^{-1} \qquad (15)$$

As was done above for the constant specific impulse mission (Eq. 12), the mission time for the constant momentum mission, t', may be explicitly written in terms of mission parameters, $\Delta' v$, m_o, P_{jet} and f':

$$t' = \frac{m_o}{2P_{jet}} (\Delta' v)^2 \frac{f'}{(1-f')} \qquad (16)$$

For the constant momentum mission at constant P_{jet}, the thrust to weight ratio, F/W, is also a constant of the mission:

$$\frac{F}{W} = \frac{2P_{jet}}{g m_o v_o}. \qquad (17)$$

The sets of three equalities expressed in Eq. 13 and Eq. 14 may be graphed as in Fig. 2a and 2b, respectively, which show the velocity increment as a function of time for missions with P_{jet}/m_o = 1 MW/20 kg = 0.05 MW/kg for mission variables B, f, and v_{jet} for the constant specific impulse

mission, and B', f', and v_o for the constant momentum mission.

Any given requirement for Δv and time, at a specified P_{jet}/m_o ratio, fixes the requirements of the other three variables. The constant specific impulse missions are shown for v_{jet} = 6275 m/s and 4500 m/s. The constant momentum

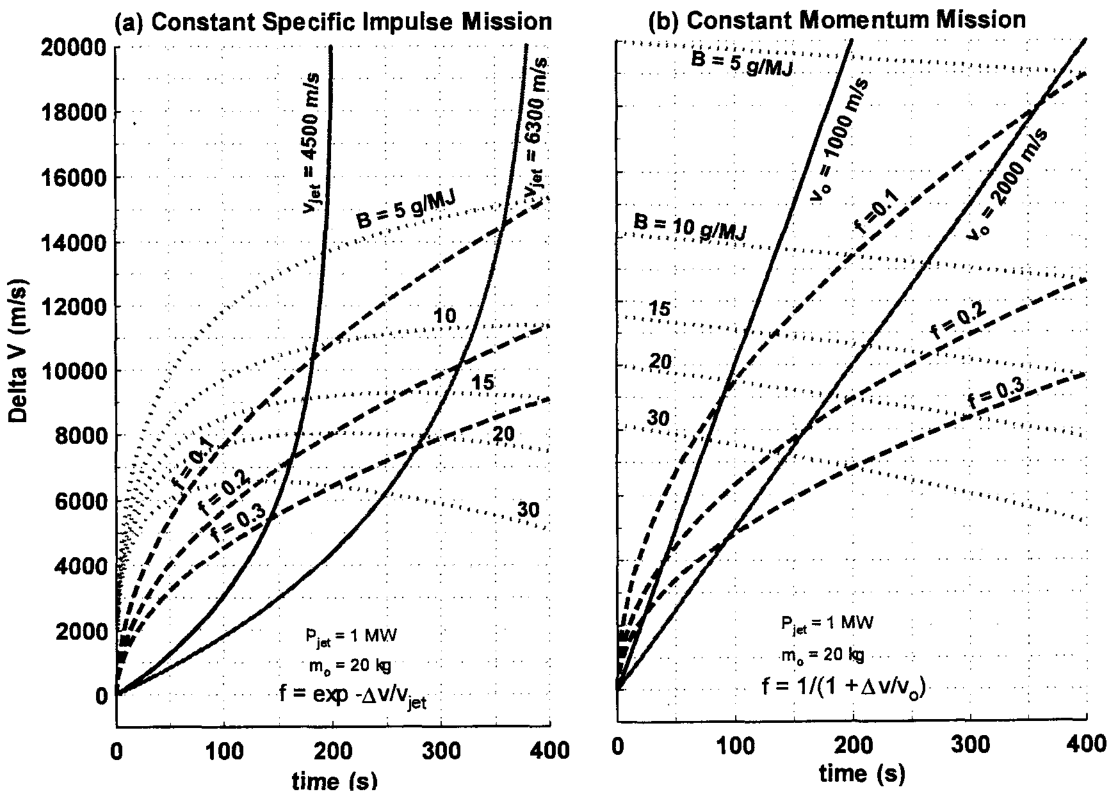

Figure 2. (a) Parameters for constant specific impulse and (b) constant momentum mission. Mission time is proportional to m_o/P_{jet}. The time axes are labeled for m_o/P_{jet} = 20 kg/MW.

missions are shown (Fig. 2b) for v_o = 2000 m/s (F/W = 5.1) and v_o = 1000 m/s (F/W = 10.2). Lines of constant mass fraction at 0.1, 0.2, and 0.3, and lines of constant m/E_{jet} at 5, 10, 15, 20, and 30 g/MJ are also shown. A constant specific impulse mission flown with P_{jet} = 1 MW and initial mass m_o = 20 kg with requirement for ΔV of 10,000 m/s, could launch a maximum of ~ 4 kg in ~ 310 seconds, with B = 12.9 g/MJ, Isp = 640 s (v_{jet} = 6275 m/s), and f = 0.203. A constant momentum mission with the same P_{jet} and m_o, but with initial velocity v_o = 2000 m/s and Δv requirement of 8000 m/s would launch ~ 4 kg in ~ 160 seconds, B' ~ 26 g/MJ, and f' = 0.2. With v_o = 3000 m/s, 6kg is propelled in ~ 200 s, and with v_o = 4000 m/s, 8 kg is propelled to 10,000 m/s in 240 s.

CONVERSION OF LASER ENERGY TO JET KINETIC ENERGY

The conversion of laser energy (E_L) to propellant (jet) kinetic energy (E_{jet}) may be analyzed in terms of the efficiencies of absorption of laser energy (β) and the efficiency of expansion of heated gas (α).

The specific internal energy of a mass m of laser heated propellant is then defined by:

$$Q^* = \beta E_L/m. \tag{18}$$

The expansion efficiency α is the efficiency of conversion of internal energy to jet kinetic energy, defined by:

$$E_{jet} = \tfrac{1}{2} m\langle v^2\rangle = \alpha m Q^* = \alpha\beta E_L, \tag{19}$$

where $\langle v^2\rangle$ is the mass-weighted average of the square of the exit velocities of the particles that comprise the jet. The momentum of the jet or the impulse imparted to a test article by the jet is given by Newton's second law, **F** = -d(mv)/dt, and may be expressed by:

$$\mathbf{I} = m\langle v \rangle, \tag{20}$$

where $\langle v \rangle$ is the mass-weighted average exit velocity of the particles that comprise the propellant. The coupling coefficient is defined by:

$$\mathbf{C} = \frac{\mathbf{I}}{E_L} \tag{21}$$

Measurements of the impulse **I** imparted to a test article when a laser pulse of energy E_L ablates a mass m of propellant have been made in numerous experiments.[9-12] Eq. 18 – Eq. 21 may be used to establish a simple and rigorous limitation on the product of **C** and $\langle v \rangle$:

$$\tfrac{1}{2}\mathbf{C}\langle \mathbf{v} \rangle = \alpha\beta\Phi \leq 1 \tag{22}$$

where Φ is the square of the ratio of the propellant's mass-weighted average velocity to its mass-weighted rms (root mean square) average velocity: $\Phi = \langle v \rangle^2/\langle v^2 \rangle$.

The laser power in a pulsed laser is

$$P_L = \omega E_L \tag{23}$$

where ω is the pulse repetition rate. The thrust is given by:

$$\mathbf{F} = \omega E_L \mathbf{C} \tag{24}$$

Thus, Eqs. (20) may be written to explicitly indicate the relationship between **F**, P_L and $\langle v \rangle$:

$$\tfrac{1}{2}\mathbf{F}\langle \mathbf{v} \rangle = \alpha\beta\Phi\, P_L \tag{25}$$

Finally, the power of the jet P_{jet}, is given by:

$$P_{jet} = \frac{1}{2}\frac{\mathbf{F}\langle \mathbf{v} \rangle}{\Phi} = \alpha\beta P_L \tag{26}$$

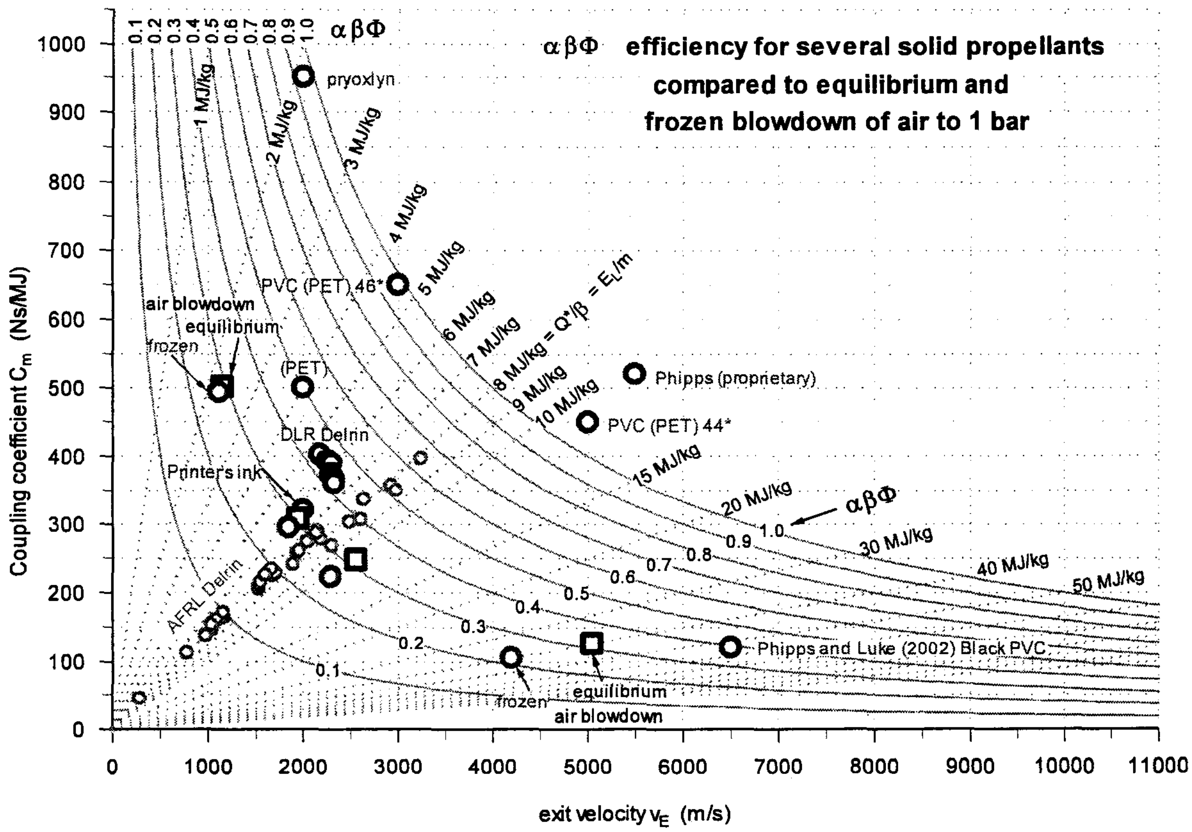

Figure 3. Fundamental relationships between coupling coefficient and jet exit velocity showing lines of constant $\alpha\beta\Phi$ and Q^*/β. Thermodynamic limits for frozen and equilibrium expansion of air and experimental points labeled "AFRL Delrin" are from Ref. 5 and 12. Experimental points labeled "DLR Delrin" are for a paraboloidal laser absorption/Delrin expansion geometry as described in Ref. 15. Other experimental points for proprietary chemically energetic ablatants are from Ref. 13.

Fig. 3 summarizes a variety of laser ablation experimental measurements in a **C** vs $\langle v\rangle$ plot with lines of constant $\alpha\beta\Phi$, and lines of constant $E_L/m = Q^*/\beta$. Fig. 3 also shows the calculated thermodynamic upper limit for equilibrium and frozen blowdown expansion of air that was heated from an initial density of STP air (1.18 kg/m^3) to specific internal energies, Q*, of 2, 6, 10, and 40 MJ/kg and expanded to 1 bar exit pressure.[5] The Figure shows the upper limit $\alpha\beta\Phi$ is ~ 0.3 for equilibrium blowdown, nearly independent of initial energy, and where Φ lies between 0.94 (low Q*) and 0.99 (high Q*). At low energy frozen blowdown produces only a slightly reduced $\alpha\beta\Phi$, but at high energy $\alpha\beta\Phi$ for frozen blowdown drops to ~ 0.2.

Laser ablation **C** vs $\langle v\rangle$ results for Delrin propellant are also shown in Fig. 3 for the AFRL lightcraft[5,9,11] and the German (DLR) paraboloid[11,12]. The largest $\alpha\beta\Phi$ values range from ~ 0.4 to 0.5. Other experimental data shown on Fig. 3 at values of $\alpha\beta\Phi$ ~ 0.9 to 1.5 were obtained from chemically energetic propellants where the chemical energy is ~ 3 times the laser energy in the case of the propellant that produces $\alpha\beta\Phi$ ~ 1.4. In the case of propellants with a chemical energy contribution, the apparent $\alpha\beta\Phi$ value is given by:

$$(\alpha\beta\Phi)_{apparent} = \alpha\Phi(\beta + m\Delta u_{chem}/E_L) \qquad (27)$$

where Δu_{chem} is the specific internal (chemical) energy of the propellant that is released during the expansion.

Φ is mathematically limited to $0 \leq \Phi \leq 1$. For a delta function distribution of velocities (i.e., a mono energetic distribution), $\Phi_{delta\ function} = 1$. For a Maxwellian gas, Φ is a mathematical constant: $\Phi_{Maxwellian\ gas} = 8/3\pi = 0.84766$. For a spherically symmetric isotropic blast wave thrusting against a flat surface, $\Phi = 0.776280$. For a Gaussian distribution of exit velocities, the relationship between the rms average velocity $v_{rms} = [\langle v^2\rangle]^{1/2}$ and the standard deviation, σ, is

$$\langle v^2\rangle = \langle v\rangle^2 + \sigma^2 \qquad (28)$$

so that Φ is given by:

$$\Phi_{Gaussian} = [1 + (\sigma/\langle v\rangle)^2]^{-1} \qquad (29)$$

Gaussian distributions with $\langle v\rangle$ = 10 km/s and σ values ranging from 0.1 to 1 km/s have Φ values between 0.9999 and 0.99. Even with relatively broad distributions with σ values that are 10% of the *arithmetic* average velocity, Φ values are greater than 0.99. For supersonic expansion, as in continuous rocket jets, velocity distributions of exit gases are shifted Maxwellians, similar to Gaussians, with Φ values > 0.999. For blowdown of hot pressurized gas from a fixed volume, Φ depends upon the initial Q*, and has been calculated for blowdown of laser heated air[5] to lie between 0.94 to 0.99 when initial pressure was 1 bar.

Sometimes ablation experiments produce plumes of heavy particle debris in addition to lightweight hot gases. When this occurs, Φ values may become very small, 0.01 or less, as may be determined by examining a bimodal velocity distribution,

$$\Phi_{bimodal} = (f_1 + f_2 r)^2/(f_1 + f_2 r^2) \qquad (30)$$

where r is the ratio of the high velocity lightweight particles (v_2) to the low velocity heavyweight particles (v_1), f_2 is the mass fraction of the lightweight particles, and f_1 is the mass fraction of the heavyweight particles. Thus, $r = v_2/v_1$ and $f_1+f_2 = 1$. For an experiment in which the mass fraction of the debris is > 0.99 and the velocity ratio is > 500, Φ < ~ 0.01. Ablation that produces a large fraction of debris has severely restricted permissible values of the product of the coupling coefficient and the average exit velocity as required by Eq. 22. Eq. 26 provides *the jet power* and explicitly includes the Φ factor. The textbooks however write Eq. 26 (and others involving E_{jet} and P_{jet}) without the Φ factor multiplying E_{jet} or P_{jet} ($\Phi = 1$ is implied), which is only valid for a mono-energetic propellant jet in which all propellant particles have the same velocity. The Φ factor arises in these equations because they contain parameters (impulse, momentum, thrust) proportional to $\langle v\rangle$ mixed with parameters (energy, power) that are proportional to $\langle v^2\rangle$. Supersonic expansions produce shifted Maxwellian distributions[4,20] that are similar to the narrow Gaussians mentioned above and have Φ values around 0.9999. Thus, in the literature and in the textbooks one does not find mention of Φ when energy quantities are compared to momentum quantities.

LASER PROPULSION MISSION STRATEGY

In principle, laser propulsion is capable of producing unit propulsive efficiency by matching the propellant exit velocity and the vehicle velocity throughout the course of the mission.

Unit propulsion efficiency occurs when the propellant is deposited in the external (inertial) frame of reference with zero velocity and all the propellant kinetic energy transfers to the vehicle. Thus, since the coupling coefficient decreases and the exit velocity increases with laser intensity (energy per area per time or power per area), the mission may be designed so that laser intensity falling on the propellant increases with altitude. Additionally, several propellants could be stacked so that high **C**, low <v> propellant is used early in the mission and low **C**, high <v> propellant is used later in the mission.

Experimental values of **C** ~ 350 Ns/MJ, <v> ~ 3000 m/s, and $\alpha\beta\Phi = 0.52$ that were measured for Delrin are shown in Fig. 3. These results were obtained with average laser intensities to 350 J/25 $cm^2/18x10^{-6}$ sec ~ 0.8 MW/cm^2. For the same $\alpha\beta\Phi$, lower intensities produce higher coupling coefficient and lower exit velocity. It is conceivable that the propellant, radiation intensity and the geometry of the vehicle absorption zone could be designed to initially produce **C** ~ 1000 N/MW, which means, with $\alpha\beta\Phi = 0.5$, that <v> ~ 1000 m/s. Manipulation of the principal <v> control parameter, Q* (<v> is proportional to the square root of Q*) could be accomplished to cause <v> to increase during the mission to match the vehicle velocity. Thus at the end of a high propulsive efficiency Earth to LEO mission, **C** ~ 125 N/MW, <v> ~ 8000 m/s.

Use of air propellant at the beginning of the Earth to LEO mission should seek to maximize the coupling coefficient and to match propellant exit velocity and vehicle velocity during the mission. It is reasonable to expect that a low temperature air plasma with 2 MJ/kg internal energy will produce a coupling coefficient **C** ~ 600 s/m (s/m = Ns/J = N/W) and $\alpha\beta\Phi$ ~ 0.3 so that the initial velocity <v> ~ 1000 m/s. Suppose the vehicle flies on air propellant to about Mach = 9 (~ 3000 m/s). At the end of this flight, higher temperature air (with internal energy ~ 20 MJ/kg, $\alpha\beta\Phi$ ~ 0.2, **C** ~ 150, <v> ~ 3000 m/s) is achievable through manipulation of the laser power and beam quality. It should be emphasized that unit propulsion efficiency is achieved when the propellant exit velocity matches the vehicle velocity. Any extra investment of energy to increase <v> beyond the vehicle velocity is a waste of laser energy and would create unnecessary high temperature stress on the vehicle's absorption zone and thrust structure.

The time scale for propellant expansion is of the order of several tens of microseconds. The time required for complete expansion and exhaustion of a pulse of propellant is ~ 200 to 300 microseconds. These time scales are the main factors that determine the optimum laser pulse width and pulse repetition rate, ω. At ω ~ 1000 Hz, and for a 250 microsecond time scale for energy absorption and complete expansion and exhaustion of the thrust structure, the duty cycle is 25%. This is a reasonable target value for pulsed laser propulsion.

In principle, the pulse width is a powerful control variable because very short pulses allow for energy deposition before the propellant expands and thus enables the highest temperatures and densities in the heated propellant. At longer pulse widths, > ~ 30 microseconds, deposition of energy in the expanding gases occurs. Thus, longer pulse widths would produce lower temperature propellant, lower <v>, and higher coupling coefficients, which is desirable for launch and the early mission times.

CONCLUSIONS

Laser propulsion is capable of accelerating small payloads through large velocity increments sufficient for launch into low earth orbit. When the ratio of laser power transmitted to the initial mass is greater than ~ 0.05 MW/kg, small payloads ~ 2 to 4 kg may be launched into low earth orbit (Δv ~ 10,000 m/s). At the same mass fraction, constant momentum propulsion launches about 23% more mass than constant specific impulse propulsion, or constant momentum propulsion launches the same mass but in about 23% less mission time.

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NOMENCLATURE (in order of usage)

Q^*	Specific energy of propellant.
v_{jet}	Propellant jet exit velocity in rocket frame of reference.
v	Velocity of propelled mass in inertial frame of reference.
α	Efficiency of conversion of internal energy to jet kinetic energy (supersonic expansion).
m	Propelled mass.
E_L	Laser energy.
Δv	Velocity change (increment) in inertial frame during a mission .
v_o	Initial velocity in inertial frame.
η_i	Instantaneous propulsion efficiency.
m_o	Initial mass.
f	Mass fraction of a mission.
x	Ratio of Δv to v_{jet} for constant specific impulse mission.
E_{jet}	Total jet kinetic energy for a mission.
B	Figure of merit for propulsion, ratio of propelled mass to E_{jet}.
P_{jet}	Kinetic power in a propellant jet.
t	time.
F	Thrust, as the thrust produced by a propellant jet.

F/W	Thrust to weight ratio.
β	fraction of laser energy absorbed by propellant.
$<v^2>$	mass-weighted average of the square of the velocities (in rocket frame of reference) of the individual particles that comprise the jet
I	Impulse.
$<v>$	mass-weighted average velocity (in rocket frame of reference) of particles that comprise the jet.
C	Coupling coefficient.
Φ	Ratio of $<v>^2$ to $<v^2>$.
P_L	Laser power.
ω	Pulse repetition rate for laser.
Δu_{chem}	Specific chemical energy in a propellant.
σ	Standard deviation of Gaussian distribution.
v_1	Velocity of slow particles in bimodal distribution.
v_2	Velocity of fast particles in bimodal distribution.
f_1	Mass fraction of slow particles in bimodal distribution.
f_2	Mass fraction of fast particles in bimodal distribution.
r	Ratio of velocities, v_2/v_1.